Dear Hu

Thank your question and good to hear that some students are struggling with difficult concepts. Now to your question.

1. I said several times that the operator gradient is meaningless if not applied on a physical quantity, scalar or vector. In the present case on a vector through cross product. But I said also that we may in some circumstances considered it as a vector. In the drawing you showed in your email this pseudo-vector lies on the plan containing vector thus it is perpendicular to current density vector you show it. The hand rule will tell that the vector potential is then vertical. Therefore, should be in opposite direction than the one you mention. Now it is true that you have an infinite number of possibilities for on the same plan. I showed you in my lecture that mathematical tricks bring physicists to agree on one, the one I showed you.
2. Now in a more formal way I also showed the analogy that you mention coming up from the relationship

And

And I told you that is to what is to . The first relation tells you that in a solenoid the direction of is determined without ambiguity with the help of the hand rule and WITHOUT considering the pseudo-vector . Please pay attention to the circulation of in a solenoid. In the case you mention knowing is enough to get the direction of exactly as in the case of a solenoid knowing is enough to get the direction of . Now simulates the circulation of in a solenoid. Therefore is as shown in my lecture.

1. Finally you have a third possibility to know exactly the value and direction of is by using the analogy with the electrostatic potential. You will learn that because of the direction of which in electrostatic simulate a static distribution of charges vertically along a wire, if is along the z-axis then and so are and the only component of the vector potential is .

I hope this helps clarifying the issue. Good luck and thank you for your interest.

Regards

Mesli